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Another proof that a finite integral domain is a field

In many books we can find proofs of the theorem on finite integral domains and all proofs run exactly as, for example, in the BIRKHOFF and McLANE'S Survey of Modern Algebra (ch. II, §3).

It may be of interest to prove the theorem in the following quite different manner.

Let R be a finite integral domain (i.e. a finite commutative ring without zero divisors) and let R^* be the set of non-zero elements of R. If $a \in R^*$, then $a, a^2, \ldots,$ all belong to R^* (there are no zero divisors in R) and because of the finitness of R we get $a^p = a^q$ for some positive integers $p, q, p > q$. We shall prove that a^{p-q} is the unit of the ring R.

In fact, if $b \in R$, we have

$$ba^p = ba^q, \; ba^p - ba^q = 0, \; (ba^{p-q} - b)\, a^q = 0.$$

But $a^q \neq 0$, so $ba^{p-q} - b = 0$, i.e. a^{p-q} is the unit of the ring R.

Now observe that for every $a \in R^*$ there is an inverse element in R^*. Indeed, if $a^p = a^q$, $p > q$, then a^{p-q} is the unit and if $p-q = 1$ there is nothing to prove, since a is the inverse for itself; if, on the other hand, $p-q > 1$, then

$$a \cdot a^{p-q-1} = a^{p-q},$$

i. e. a^{p-q-1} is the inverse of a.

Thus the ring R is a field.

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NOWY DOWÓD TWIERDZENIA, ŻE SKOŃCZONY PIERŚCIEŃ CAŁKOWITY JEST CIAŁEM.

Streszczenie

Podano nowy dowód twierdzenia wymienionego w tytule

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